

M3 - January 2007

1- a) At $x=0$, $a=0$ so max speed is reached

b) $a = \frac{1}{12}(30-x)$

$$\frac{v dv}{dx} = \frac{1}{12}(30-x)$$

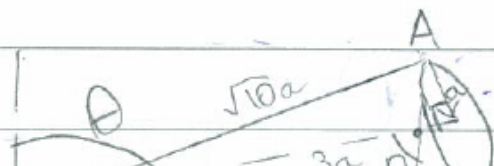
$$\int_{10}^v v dv = \frac{1}{12} \int_{30}^x (30-x) dx$$

$$\frac{1}{2}[v^2]_{10}^v = \frac{1}{6} \left[30x - \frac{1}{2}x^2 \right]_{30}^x$$

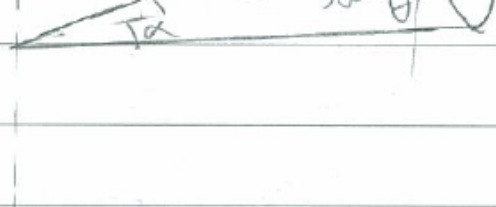
$$v^2 - 100 = \frac{1}{6} \left(30x - \frac{1}{2}x^2 - 900 + 450 \right)$$

$$v^2 = -\frac{1}{12}x^2 + 5x + 25$$

2-



~~tan theta = a/2~~ $\tan \theta = \frac{a}{2} = \frac{4}{2}$



$$\frac{3a}{4}$$

$$\theta = 53.1^\circ \text{ (dp)}$$

3- a) $F = \frac{1 \times 3}{2a} = \frac{3.6mg \times a^2}{2 \times a \times 3^2} = \frac{mg a}{5}$

b) $\uparrow mg = R$
 $\rightarrow \mu R$
 $= \mu mg$

$M \dot{v}_0 = m \dot{v}_A + W + D$

$\frac{1}{2} m 2v = \frac{mg a}{5} + \frac{4}{3} \mu mg$

$1 = \frac{1}{5} + \frac{4}{3} \mu$

$\frac{5}{5} = \frac{1}{5} + \frac{4}{3} \mu$

$\frac{3}{5} = \mu$

4. a) $m v_B = m v_D$

$$\frac{1}{2} m 3ag = \frac{1}{2} m v^2 + m g (a + a \cos \theta)$$

$$3ag = v^2 + 2ga + 2ga \cos \theta$$

$$v^2 = ag - 2ag \cos \theta$$

b) $[F = ma]_k$

$$T + mg \cos \theta = \frac{m v^2}{r}$$

$$T = \frac{m}{a} (ag - 2ag \cos \theta) - mg \cos \theta$$

$$= mg - 2mg \cos \theta - mg \cos \theta$$

$$= mg - 3mg \cos \theta = (1 - 3 \cos \theta) mg$$

$$c) 0 = (1 - 3\cos\theta)mg$$
$$\rightarrow 0 = 1 - 3\cos\theta$$
$$\cos\theta = \frac{1}{3}$$

$$\text{Height} = a + a\cos\theta$$
$$= a + \frac{1}{3}a = \frac{4}{3}a$$

$$d) v^2 = ay - 2ay \times \frac{1}{3} = ay - \frac{2}{3}ay = \frac{1}{3}ay$$
$$v = \sqrt{\frac{1}{3}ay}$$

$$v_x = \sqrt{\frac{1}{3}ay} \cos\theta = \frac{1}{3} \sqrt{\frac{1}{3}ay}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_x^2 + mgh$$

$$\frac{1}{6}ay = \frac{1}{2} \cdot \frac{1}{9} \cdot \frac{1}{3}ay + gh$$

$$h = \frac{a}{6} - \frac{a}{54} = \frac{4a}{27}$$

$$S. a) \uparrow mg = T \cos \theta$$

$$\tan \theta = \frac{r}{h}$$

$$r = h \tan \theta$$

$$\leftarrow [F = ma]$$

$$T + T \sin \theta = m \omega^2 r$$

$$\frac{mg}{\cos \theta} + \frac{mg \sin \theta}{\cos \theta} = m \omega^2 h \frac{\sin \theta}{\cos \theta}$$

$$g + g \sin \theta = \omega^2 h \sin \theta$$

$$g(1 + \sin \theta) = \omega^2 h \sin \theta$$

$$\omega^2 = \frac{g}{h} \left(\frac{1 + \sin \theta}{\sin \theta} \right)$$

$$b) \frac{h \omega^2}{g} = \frac{1 + \sin \theta}{\sin \theta}$$

$$\frac{1 + \sin \theta}{\sin \theta} = \frac{1}{\sin \theta} + 1$$

$$\therefore \frac{1}{\sin \theta} + 1 > 2$$

$$\frac{h \omega^2}{g} > 2$$

$$\frac{1 + \sin \theta}{\sin \theta}$$

$$c) \omega = \sqrt{\frac{3g}{h}}$$

$$\omega^2 = \frac{3g}{h}$$

$$\frac{3g}{4} = \frac{g}{4} \left(\frac{1 + \sin \theta}{\sin \theta} \right)$$

$$3 \sin \theta = 1 + \sin \theta$$

$$\sin \theta = \frac{1}{2}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 - \frac{1}{4} = \cos^2 \theta$$

$$\cos^2 \theta = \frac{3}{4}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

Q. a) $V = \int_1^2 \pi y^2 dx = \int_1^2 \frac{\pi}{4} x^{-4} dx$

$$= \frac{\pi}{4} \left[-\frac{1}{3} x^{-3} \right]_1^2 = \frac{\pi}{4} \left(-\frac{1}{24} + \frac{1}{3} \right)$$

$$= \frac{7\pi}{96}$$

$$V(1+d) = \int_{1/4}^2 \frac{\pi}{4} x^{-4} dx = \frac{\pi}{4} \left[-\frac{1}{3} x^{-3} \right]_{1/4}^2$$

$$\frac{7\pi}{96} + \frac{7\pi}{96} d = \frac{\pi}{4} \left(-\frac{1}{8} + \frac{1}{2} \right) = \frac{3\pi}{32}$$

$$d = \left(\frac{3}{32} - \frac{7}{96} \right) \div \frac{7}{96} = \frac{7}{96} \div \frac{7}{96} = 1 \text{ m}$$

$$T = \frac{mg}{\cos \theta}$$

$$= \frac{2mg}{\sqrt{3}}$$

$$b) \frac{7\pi}{96} \times \frac{5}{7} + \left(1 + \frac{1}{2} \times \frac{3}{8}\right) \frac{2}{3} \pi \left(\frac{1}{2}\right)^3 = \left[\frac{7\pi}{96} + \frac{2}{3} \pi \left(\frac{1}{2}\right)^3\right] \bar{d}$$

$$\frac{5\pi}{96} + \frac{19}{168} \cdot \frac{21}{3} \cdot \frac{1}{8} \pi = \left(\frac{7\pi}{96} + \frac{1}{12} \pi\right) \bar{d}$$

$$\frac{5}{96} + \frac{19}{192} = \frac{5}{32} \bar{d}$$

$$\bar{d} = \frac{32 \times 29}{192 + 5} = \frac{29}{30} \text{ m}$$

$$F = \uparrow mg = T$$

$$T = 0.25g$$

$$= 2.45$$

$$T = \frac{\lambda x}{a}$$

$$2.45 = \frac{0.05 \lambda}{0.8}$$

$$\lambda = \frac{0.8 \times 2.45}{0.05} = 39.2$$

$$b) T = \lambda x - 39.2 \times (0.05 + x) = 2.45 + 49x$$

$\frac{1}{a} = \frac{1}{0.8}$

$\downarrow (F = ma)$

$mg - T = m\ddot{x}$

$0.25 \times 9.8 - 2.45 - 49x = 0.25\ddot{x}$

$2.45 - 2.45 - 49x = 0.25\ddot{x}$

$\ddot{x} = -196x$

\therefore SHM with $\omega^2 = 196$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{14} = \frac{\pi}{7}$

c) $v^2 = \omega^2(a^2 - x^2)$

$v^2 = 196(0.1^2 - 0.05^2) = 1.47$

$v = 1.21 \text{ ms}^{-1} \text{ (3sf)}$

d) $x = 0.1 \cos 14t$
 $-0.05 = 0.1 \cos 14t$

~~0.1 \cos 14t = -0.05~~
~~\cos 14t = -0.5~~
~~14t = \cos^{-1}(-0.5)~~
~~t = \frac{\cos^{-1}(-0.5)}{14}~~
~~t = \frac{2.094}{14}~~
~~t = 0.1496~~

$\cos 14t = -0.5$ [v = total]

$t = \frac{\pi}{7}$

$0 = 1.21 \pm 9.8t$
 $t = 0.124$

Total time:

$\frac{\pi}{21} + 0.124$

~~0.157~~

$= 0.273 \text{ s}$